Multilinear Galois Mode (MGM)
draft-smyshlyaev-mgm-10

Abstract

Multilinear Galois Mode (MGM) is an authenticated encryption with associated data block cipher mode based on EtM principle. MGM is defined for use with 64-bit and 128-bit block ciphers.

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1. Introduction

Multilinear Galois Mode (MGM) is an authenticated encryption with associated data block cipher mode based on EtM principle. MGM is defined for use with 64-bit and 128-bit block. The MGM design principles can easily be applied to other block sizes.

1.1. Existing Constructions

The text will be added in the future versions of the draft.

2. Conventions Used in This Document

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [RFC2119].

3. Basic Terms and Definitions

This document uses the following terms and definitions for the sets and operations on the elements of these sets:

\[ V^* \] the set of all bit strings of a finite length (hereinafter referred to as strings), including the empty string; substrings and string components are enumerated from right to left starting from zero;

\[ V_s \] the set of all bit strings of length s, where s is a non-negative integer;
the bit length of the bit string X (if X is an empty string, then |X| = 0);

X || Y concatenation of strings X and Y both belonging to V*, i.e., a string from V_(|X| + |Y|), where the left substring from V_(|X|) is equal to X, and the right substring from V_(|Y|) is equal to Y;

a^s the string in V_s that consists of s ‘a’ bits: a^s = (a, a, ... , a), ‘a’ in V_1;

(xor) exclusive-or of the two bit strings of the same length,

Z_(2^s) ring of residues modulo 2^s;

MSB_i: V_s -> V_i the transformation that maps the string X = (x_{s-1}, ... , x_0) in V_s into the string MSB_i(X) = (x_{s-1}, ... , x_{s-i}) in V_i, i <= s, (most significant bits);

Int_s: V_s -> Z_{2^s} the transformation that maps a string X = (x_{s-1}, ... , x_0) in V_s into the integer Int_s(X) = 2^{s-1} * x_{s-1} + ... + 2 * x_1 + x_0 (the interpretation of the bit string as an integer);

Vec_s: Z_{2^s} -> V_s the transformation inverse to the mapping Int_s (the interpretation of an integer as a bit string);

E_K: V_n -> V_n the block cipher permutation under the key K in V_k;

k the bit length of the block cipher key;

n the block size of the block cipher (in bits);

len: V_s -> V_{n/2} the transformation that maps a string X in V_s, 0 <= s <= 2^(n/2) - 1, into the string len(X) = Vec_{n/2}(|X|) in V_{n/2}, where n is the block size of the used block cipher;

[+] the addition operation in Z_(2^{n/2}), where n is the block size of the used block cipher;

(x) multiplication in GF(2^n), where n is the block size of the used block cipher; if n = 64, then the field polynomial is equal to f = x^64 + x^4 + x^3 + x + 1; if n = 128, then the field polynomial is equal to f = x^128 + x^7 + x^2 + x + 1;
 incr\_l: V\_n \rightarrow V\_n the transformation that maps a string L || R, where L, R in V\_{n/2}, into the string incr\_l(L || R) = Vec\_{n/2}(Int\_{n/2}(L) \[+\] 1) || R;

 incr\_r: V\_n \rightarrow V\_n the transformation that maps a string L || R, where L, R in V\_{n/2}, into the string incr\_r(L || R) = L || Vec\_{n/2}(Int\_{n/2}(R) \[+\] 1).

4. Specification

An additional parameter that defines the functioning of MGM mode is the bit length S of the authentication tag, 32 \leq S \leq 128. The value of S MUST be fixed for a particular protocol. The choice of the value S involves a trade-off between message expansion and the forgery probability.

4.1. MGM Encryption and Authentication Procedure

The MGM encryption and authentication procedure takes the following parameters as inputs:

1. Encryption key K in V\_k.
2. Initial counter nonce ICN in V\_{n-1}.
3. Plaintext P, 0 \leq |P| < 2^{n/2}. If |P| > 0, then P = P\_1 || \ldots || P\_q, P\_i in V\_n, i = 1, \ldots, q - 1, P\_q in V\_u, 1 \leq u \leq n. If |P| = 0, then by definition P\_q is empty, and the q and u parameters are set as follows: q = 0, u = n.

4. Associated authenticated data A, 0 \leq |A| < 2^{n/2}. If |A| > 0, then A = A\_1 || \ldots || A\_h, A\_j in V\_n, j = 1, \ldots, h - 1, A\_h in V\_t, 1 \leq t \leq n. If |A| = 0, then by definition A\_h is empty, and the h and t parameters are set as follows: h = 0, t = n. The associated data is authenticated but is not encrypted.

The MGM encryption and authentication procedure outputs the following parameters:

1. Initial counter nonce ICN.
2. Associated authenticated data A.
3. Ciphertext C in V\_{|P|}.
4. Authentication tag T in V\_S.
The MGM encryption and authentication procedure consists of the following steps:

1. **Encryption step:**
   - \( Y_1 = E_K(0 \ || \ ICN) \),
   - For \( i = 2, 3, \ldots, q \) do
     \( Y_i = \text{incr}_r(Y_{i-1}) \),
   - For \( i = 1, 2, \ldots, q - 1 \) do
     \( C_i = P_i \ (\text{xor} \ E_K(Y_i)) \),
   - \( C_*_q = P*_q \ (\text{xor} \ \text{MSB}_u(E_K(Y_q))) \),
   - \( C = C_1 \ || \ ... \ || C_*_q \).

2. **Padding step:**
   - \( A_h = A*_h \ || \ 0^{n-t} \),
   - \( C_q = C_*_q \ || \ 0^{n-u} \).

3. **Authentication tag \( T \) generation step:**
   - \( Z_1 = E_K(1 \ || \ ICN) \),
   - \( \text{sum} = 0 \),
   - For \( i = 1, 2, \ldots, h \) do
     \( H_i = E_K(Z_i) \),
     \( \text{sum} = \text{sum} \ (\text{xor}) \ (H_i \ (x) A_i) \),
     \( Z_{i+1} = \text{incr}_l(Z_i) \),
   - For \( j = 1, 2, \ldots, q \) do
     \( H_{(h+j)} = E_K(Z_{(h+j)}) \),
     \( \text{sum} = \text{sum} \ (\text{xor}) \ (H_{(h+j)} \ (x) C_j) \),
     \( Z_{(h+j+1)} = \text{incr}_l(Z_{(h+j)}) \),
   - \( H_{(h+q+1)} = E_K(Z_{(h+q+1)}) \),
   - \( T = \text{MSB}_S(E_K(\text{sum} \ (\text{xor}) H_{(h+q+1)} \ (x) \ (\text{len}(A) \ || \ \text{len}(C)))) \).

4. Return \((ICN, A, C, T)\).

The ICN value for each message that is encrypted under the given key \( K \) must be chosen in a unique manner. Using the same ICN values for two different messages encrypted with the same key eliminates the security properties of this mode.

Users who do not wish to encrypt plaintext can provide a string \( P \) of zero length. Users who do not wish to authenticate associated data can provide a string \( A \) of zero length. The length of the associated
data $A$ and of the plaintext $P$ MUST be such that $0 < |A| + |P| < 2^{n/2}$.

### 4.2 MGM Decryption and Authentication Check Procedure

The MGM decryption and authentication procedure takes the following parameters as inputs:

1. The encryption key $K$ in $V_k$.
2. The initial counter nonce $ICN$ in $V_{n-1}$.
3. The associated authenticated data $A$, $0 \leq |A| < 2^{n/2}$. $A = A_{-1} || \ldots || A_h$, $A_j$ in $V_n$, $j = 1, \ldots, h - 1$, $A_h$ in $V_t$, $1 \leq t \leq n$.
4. The ciphertext $C$, $0 \leq |C| < 2^{n/2}$. $C = C_{-1} || \ldots || C_q$, $C_i$ in $V_n$, $i = 1, \ldots, q - 1$, $C_q$ in $V_u$, $1 \leq u \leq n$.
5. The authenticated tag $T$ in $V_S$.

The MGM decryption and authentication procedure outputs FAIL or the following parameters:

1. Plaintext $P$ in $V_{|C|}$.
2. Associated authenticated data $A$.

The MGM decryption and authentication procedure consists of the following steps:
MGM-Decrypt(K, ICN, A, C, T)

1. Padding step:
   - A_h = A*_h || 0^(n-t),
   - C_q = C*_q || 0^(n-u).

2. Authentication tag T verification step:
   - Z_1 = E_K(1 || ICN),
   - sum1 = 0, sum2 = 0,
   - For i = 1, 2, ..., h do
     H_i = E_K(Z_i),
     sum1 = sum1 (xor) ( H_i (x) A_i ),
     Z_{i+1} = incr_l(Z_i),
   - For j = 1, 2, ..., q do
     H_{h+j} = E_K(Z_{h+j}),
     sum2 = sum2 (xor) ( H_{h+j} (x) C_j ),
     Z_{h+j+1} = incr_l(Z_{h+j}),
   - H_{h+q+1} = E_K(Z_{h+q+1}),
   - T' = MSB_S(E_K(sum1 (xor) sum2 (xor)
     H_{h+q+1} (x) (len(A) || len(C)))),
   - If T' != T then return FAIL
   return FAIL.

3. Decryption step:
   - Y_1 = E_K(0 || ICN),
   - For i = 2, 3, ..., q do
     Y_i = incr_r(Y_{i-1}),
   - For i = 1, 2, ..., q - 1 do
     P_i = C_i (xor) E_K(Y_i),
   - P_q = C_q (xor) MSB_u(E_K(Y_q)),
   - P = P_1 || ... || P_q.


5. Rationale

The MGM mode was originally proposed in [PDMODE].

From the operational point of view the MGM mode is designed to be parallelizeable, inverse free, online and to provide availability of precomputations.

Parallelizability of the MGM mode is achieved due to its counter-type structure and the usage of the multilinear function for authentication. Indeed, both encryption blocks E_K(Y_i) and
authentication blocks $H_i$ are produced in the counter mode manner, and the multilinear function determined by $H_i$ is parallelizable in itself. Additionally, the counter-type structure of the mode provides the inverse free property.

The online property means the possibility to process message even if it is not completely received (so its length is unknown). To provide this property the MGM mode uses blocks $E_K(Y_i)$ and $H_i$ which are produced basing on two independent source blocks $Y_i$ and $Z_i$.

Availability of precomputations for the MGM mode means the possibility to calculate $H_i$ and $E_K(Y_i)$ even before data is retrieved. It is holds due to again the usage of counters for calculating them.

The MGM mode incorporates some mechanisms for advancing cryptographic properties. Further we note the main ones:

Different functions generating the counter values: The functions $incr_r$ and $incr_l$ are chosen to minimize intersection (if it happens) between the sets of counter values $Y_i$ and $Z_i$.

Ciphering of the multilinear function output: This procedure allows to resist attacks based on padding and linear properties (see [Ferg05] for details).

Multilinear function for authentication: It allows to resist the small subgroup attacks [Saar12].

Ciphering of the nonces $(0 || ICN)$ and $(1 || ICN)$: The aim of this ciphering is to minimize the number of plaintext/ciphertext pairs of blocks known to an adversary. Small number of these pairs allows to resist attacks that need substantial amount of such material (e.g., linear and differential cryptanalysis, side-channel attacks).

6. References

6.1. Normative References


6.2. Informative References


Appendix A. Test Vectors

Test vectors for the Kuznyechik block cipher (n = 128, k = 256) defined in [GOST3412-2015] (the English version can be found in [RFC7801]).

Encryption key K:
00000: 88 99 AA BB CC DD EE FF 00 11 22 33 44 55 66 77
00010: FE DC BA 98 76 54 32 10 01 23 45 67 89 AB CD EF

Associated authenticated data A:
00000: 02 02 02 02 02 02 02 02 01 01 01 01 01 01 01 01
00010: 04 04 04 04 04 04 04 04 03 03 03 03 03 03 03 03
00020: EA 05 05 05 05 05 05 05 05 05

Plaintext P:
00000: 11 22 33 44 55 66 77 00 FF EE DD CC BB AA 99 88
00010: 00 11 22 33 44 55 66 77 88 99 AA BB CC EE FF 0A
00020: 11 22 33 44 55 66 77 88 99 AA BB CC EE FF 0A 00
00030: 22 33 44 55 66 77 88 99 AA BB CC EE FF 0A 00 11
00040: AA BB CC

1. Encryption step:

0^1 || ICN:
00000: 11 22 33 44 55 66 77 00 FF EE DD CC BB AA 99 88

Y_1:
00000: 7F 67 9D 90 BE BC 24 30 5A 46 8D 42 B9 D4 ED CD

E_K(Y_1):
00000: B8 57 48 C5 12 F3 19 90 AA 56 7E F1 53 35 DB 74

Y_2:
00000: 7F 67 9D 90 BE BC 24 30 5A 46 8D 42 B9 D4 ED CE
E_K(Y_2):
00000: 80 64 F0 12 6F AC 9B 2C 5B 6E AC 21 61 2F 94 33

Y_3:
00000: 7F 67 9D 90 BE BC 24 30 5A 46 8D 42 B9 D4 ED CF
E_K(Y_3):
00000: 58 58 82 1D 40 C0 CD 0D 0A C1 E6 C2 47 09 8F 1C

Y_4:
00000: 7F 67 9D 90 BE BC 24 30 5A 46 8D 42 B9 D4 ED D0
E_K(Y_4):
00000: E4 3F 50 81 B5 8F 0B 49 01 2F 8E E8 6A CD 6D FA

Y_5:
00000: 7F 67 9D 90 BE BC 24 30 5A 46 8D 42 B9 D4 ED D1
E_K(Y_5):
00000: 86 CE 9E 2A 0A 12 25 E3 33 56 91 B2 0D 5A 33 48

C:
00000: A9 75 7B 81 47 95 6E 90 55 B8 A3 3D E8 9F 42 FC
00010: 80 75 D2 21 2B F9 FD 5B D3 F7 06 9A AD C1 6B 39
00020: 49 7A B1 59 15 A6 BA 85 93 6B 5D 0E A9 F6 85 1C
00030: C6 0C 14 D4 D3 F8 83 D0 AB 94 42 06 95 C7 6D EB
00040: 2C 75 52

2. Padding step:
A_1 || ... || A_h:
00000: 02 02 02 02 02 02 02 02 02 02 02 01 01 01 01 01 01 01 01
00010: 04 04 04 04 04 04 04 04 04 04 04 03 03 03 03 03 03 03 03
00020: EA 05 05 05 05 05 05 05 05 05 05 05 05 05 05 05 05 05 05

C_1 || ... || C_q:
00000: A9 75 7B 81 47 95 6E 90 55 B8 A3 3D E8 9F 42 FC
00010: 80 75 D2 21 2B F9 FD 5B D3 F7 06 9A AD C1 6B 39
00020: 49 7A B1 59 15 A6 BA 85 93 6B 5D 0E A9 F6 85 1C
00030: C6 0C 14 D4 D3 F8 83 D0 AB 94 42 06 95 C7 6D EB
00040: 2C 75 52 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00

3. Authentication tag T generation step:
1^1 || ICN:
00000: 91 22 33 44 55 66 77 00 FF EE DD CC BB AA 99 88
Z_1:
00000: 7F C2 45 A8 58 6E 66 02 A7 BB DB 27 86 BD C6 6F
H_1:
00000: 8D B1 87 D6 53 83 0E A4 BC 44 64 76 95 2C 30 0B
current sum: 00000: 4C F4 27 F4 AD B7 5C F4 C0 DA 39 D5 AB 48 CF 38

Z_2:
00000: 7F C2 45 A8 58 6E 66 03 A7 BB DB 27 86 BD C6 6F
H_2:
00000: 7A 24 F7 26 30 E3 76 37 21 C8 F3 CD B1 DA 0E 31
current sum: 00000: 94 95 44 0E F6 24 A1 DD C6 F5 D9 77 28 50 C5 73

Z_3:
00000: 7F C2 45 A8 58 6E 66 04 A7 BB DB 27 86 BD C6 6F
H_3:
00000: 44 11 96 21 17 D2 06 35 C5 25 E0 A2 4D B4 B9 0A
current sum: 00000: A4 9A 8C D8 A6 F2 74 23 DB 79 E4 4A B3 06 D9 42

Z_4:
00000: 7F C2 45 A8 58 6E 66 05 A7 BB DB 27 86 BD C6 6F
H_4:
00000: D8 C9 62 3C 4D BF E8 14 CE 7C 1C 0C EA A9 59 DB
current sum: 00000: 09 FE 3F 6A 83 3C 21 B3 90 27 D0 20 6A 84 E1 5A

Z_5:
00000: 7F C2 45 A8 58 6E 66 06 A7 BB DB 27 86 BD C6 6F
H_5:
00000: A5 E1 F1 95 33 3E 14 82 96 99 31 BF BE 6D FD 43
current sum: 00000: B5 DA 26 BB 00 EB A8 04 35 D7 97 6B C6 B5 46 4D

Z_6:
00000: 7F C2 45 A8 58 6E 66 07 A7 BB DB 27 86 BD C6 6F
H_6:
00000: B4 CA 80 8C AC CF B3 F9 17 24 E4 8A 2C 7E E9 D2
current sum: 00000: DD 1C 0E EE F7 83 C8 EB 2A 33 F3 58 D7 23 0E E5

Z_7:
00000: 7F C2 45 A8 58 6E 66 08 A7 BB DB 27 86 BD C6 6F
H_7:
00000: 72 90 8F C0 74 E4 69 E8 90 1B D1 88 EA 91 C3 31
current sum: 00000: 89 6C E1 08 32 EB EA F9 06 9F 3F 73 76 59 4D 40
\[ Z_{8}: \]
\[ 00000: \quad 7F \ C2 \ 45 \ A8 \ 58 \ 6E \ 66 \ 09 \ A7 \ BB \ DB \ 27 \ 86 \ BD \ C6 \ 6F \]
\[ H_{8}: \]
\[ 00000: \quad 23 \ CA \ 27 \ 15 \ B0 \ 2C \ 6B \ 31 \ 3B \ FD \ AC \ B3 \ 9E \ 4D \ 0F \ B8 \]
\[ \text{current sum:} \]
\[ 00000: \quad 99 \ 1A \ F5 \ C9 \ D0 \ 80 \ F7 \ 6D \ 87 \ FE \ 64 \ 9E \ 7C \ 93 \ C6 \ 42 \]
\[ Z_{9}: \]
\[ 00000: \quad 7F \ C2 \ 45 \ A8 \ 58 \ 6E \ 66 \ 0A \ A7 \ BB \ DB \ 27 \ 86 \ BD \ C6 \ 6F \]
\[ H_{9}: \]
\[ 00000: \quad BC \ BC \ E6 \ C4 \ 1A \ A3 \ 55 \ A4 \ 14 \ 88 \ 62 \ BF \ 64 \ BD \ 83 \ 0D \]
\[ \text{len(A) } || \text{ len(C):} \]
\[ 00000: \quad 00 \ 00 \ 00 \ 00 \ 00 \ 00 \ 01 \ 48 \ 00 \ 00 \ 00 \ 00 \ 00 \ 02 \ 18 \]
\[ \text{sum (xor) } H_{9} \ (x) \ \text{(len(A) } || \text{ len(C))}: \]
\[ 00000: \quad C0 \ C7 \ 22 \ DB \ 5E \ 0B \ D6 \ DB \ 25 \ 76 \ 73 \ 83 \ 3D \ 56 \ 71 \ 28 \]

Tag T:
\[ 00000: \quad CF \ 5D \ 65 \ 6F \ 40 \ C3 \ 4F \ 5C \ E8 \ BB \ 0E \ 29 \ FC \ DB \ 4C \]

**Appendix B. Contributors**

- Evgeny Alekseev
  CryptoPro
  alekseev@cryptopro.ru

- Ekaterina Smyshlyaeva
  CryptoPro
  ess@cryptopro.ru

- Lilia Akhmetzyanova
  CryptoPro
  lah@cryptopro.ru

- Grigory Marshalko
  TC 26
  marshalko_gb@tc26.ru

- Vladimir Rudskoy
  TC 26
  rudskoy_vi@tc26.ru

- Alexey Nesterenko
  National Research University Higher School of Economics
  anesterenko@hse.ru
Authors’ Addresses

Stanislav Smyshlyaev (editor)
CryptoPro
Phone: +7 (495) 995-48-20
Email: svs@cryptopro.ru

Vladislav Nozdrunov
TC 26
Email: nozdrunov_vi@tc26.ru

Vasily Shishkin
TC 26
Email: shishkin_va@tc26.ru